

DERIVARE

$$y = \left(\frac{3}{x}\right)^{2 + \sin x} + 3\sqrt{x} + 6 \quad \checkmark$$

STUDIO FUNZIONE

$$y = 3\sqrt[3]{x^2 - 8} + 2 \quad \checkmark$$

INTEGRARE

$$\int_0^{\pi} [\sin 2x + x \sin 3x + 2x^3 - 3] dx \quad \checkmark$$

LIMITI

$$\lim_{x \rightarrow +\infty} \sqrt{2x^3 + 1} - \sqrt{x^3} \quad \checkmark$$

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$$y = \left(\frac{3}{x}\right)^{2+5\ln x} + \underbrace{3\sqrt{x}}_{\text{facile}} + \underbrace{6}_{\text{facile}}$$

Calcolo separatamente la derivata di

$$y = \left(\frac{3}{x}\right)^{2+5\ln x}$$

Applico il ln (logaritmo naturale) ad entrambi i membri:

$$\lg(y) = \lg\left(\frac{3}{x}\right)^{2+5\ln x} \quad \text{PROP. LOGARITMI} \quad \lg a^m = m \cdot \lg a$$

$$\lg y = (2+5\ln x) \cdot \lg\left(\frac{3}{x}\right) \quad \text{DERIVO ENTRAMBI I MEMBRI RISPETTO AD } x$$

$$\frac{1}{y} \cdot y' = \frac{5}{x} \cdot \lg\left(\frac{3}{x}\right) + (2+5\lg x) \cdot \frac{1}{\frac{3}{x}} \cdot 3 \cdot \left(-\frac{1}{x^2}\right)$$

$$\frac{y'}{y} = \frac{5}{x} \lg\left(\frac{3}{x}\right) + (2+5\lg x) \frac{x}{3} \cdot \left(-\frac{3}{x^2}\right)$$

$$\frac{y'}{y} = \frac{5}{x} \lg\left(\frac{3}{x}\right) - \frac{2+5\lg x}{x} \quad \text{moltiplico entrambi i membri per } y$$

$$y' = \left(\frac{5}{x} \lg\left(\frac{3}{x}\right) - \frac{2+5\lg x}{x}\right) \cdot y$$

$$y' = \left(\frac{5}{x} \lg\left(\frac{3}{x}\right) - \frac{2+5\lg x}{x}\right) \cdot \left(\frac{3}{x}\right)^{2+5\lg x}$$

$$y' = \left(\frac{5}{x} \lg\left(\frac{3}{x}\right) - \frac{2+5\lg x}{x}\right) \left(\frac{3}{x}\right)^{2+5\lg x} + \frac{3}{2\sqrt{x}}$$

$$(1) \int x \cos 2x \, dx = \left\| \begin{array}{l} 2x = t \\ \frac{2dx}{2} = \frac{dt}{2} \end{array} \right\| = \frac{1}{2} \int x \cos t \, dt = \left(-\frac{1}{2} \cos t = -\frac{1}{2} \cos 2x + C \right)$$

VERIFICATION $\left(-\frac{1}{2} \cos 2x\right)' = -\frac{1}{2} (-\sin 2x) \cdot 2 = +\sin 2x \checkmark$

$$(2) \int x \sin 3x \, dx = -\frac{1}{3} x \cos 3x - \int -\frac{1}{3} \cos 3x \, dx = -\frac{1}{3} x \cos 3x + \frac{1}{3} \int \cos 3x \, dx$$

$$= -\frac{1}{3} x \cos 3x + \frac{1}{3} \cdot \frac{1}{3} \sin 3x = -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x$$

VERIFICATION

$$\left(-\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x\right)' = -\frac{1}{3} (\cos 3x + x (-\sin 3x) \cdot 3) + \frac{1}{9} \cos 3x \cdot 3 =$$

$$= -\frac{1}{3} \cos 3x + \frac{1}{3} x \sin 3x + \frac{1}{3} \cos 3x = x \sin 3x \checkmark$$

$$(3) \int 2x^3 \, dx = 2 \int x^3 \, dx = 2 \frac{x^4}{4} = \left(\frac{x^4}{2} + C \right)$$

VERIFICATION $\left(\frac{x^4}{2}\right)' = \frac{4x^3}{2} = 2x^3 \checkmark$

$$(4) \int -3 \, dx = -3 \int dx = (-3x) \checkmark$$

RESULTADO $= \left[(1) + (2) + (3) + (4) \right]_0^{\pi} = 0$

$$= \left[-\frac{1}{2} \cos 2x - \frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + \frac{x^4}{2} - 3x \right]_0^{\pi} =$$

$$= \left(-\frac{1}{2} \cos 2\pi - \frac{1}{3} \pi \cos 3\pi + \frac{1}{9} \sin 3\pi + \frac{\pi^4}{2} - 3\pi \right) - \left(-\frac{1}{2} \cos 2 \cdot 0 - \frac{1}{3} 0 \cdot \cos 3 \cdot 0 + \frac{1}{9} \sin 3 \cdot 0 + \frac{0^4}{2} - 3 \cdot 0 \right) =$$

$$= \left(-\frac{1}{2} \cdot 1 - \frac{1}{3} \pi (-1) + \frac{1}{9} \cdot 0 + \frac{\pi^4}{2} - 3\pi \right) - \left(-\frac{1}{2} \cos 0 - 0 + \frac{1}{9} \sin 0 + 0 - 0 \right) =$$

$$= \left(-\frac{1}{2} + \frac{\pi}{3} + 0 + \frac{\pi^4}{2} - 3\pi \right) - \left(-\frac{1}{2} \cdot 1 - 0 + \frac{1}{9} \cdot 0 \right) =$$

$$= \cancel{-\frac{1}{2}} + \frac{\pi}{3} + \frac{\pi^4}{2} - 3\pi + \cancel{\frac{1}{2}} = \frac{\pi}{3} + \frac{\pi^4}{2} - 3\pi = \frac{\pi^4}{2} + \pi \left(\frac{1}{3} - 3 \right) = \frac{\pi^4}{2} + \pi \left(-\frac{8}{3} \right) =$$

$$= \frac{\pi^4}{2} - \frac{8\pi}{3} \quad 40,32$$

43,70

$$\begin{aligned} \lim_{x \rightarrow +\infty} \sqrt{2x^3+1} - \sqrt{x^3} &= \lim_{x \rightarrow +\infty} \sqrt{x^3 \left(2 + \frac{1}{x^3}\right)} - \sqrt{x^3} = \lim_{x \rightarrow +\infty} \sqrt{x^3} \sqrt{2 + \frac{1}{x^3}} - \sqrt{x^3} = \\ &= \lim_{x \rightarrow +\infty} \sqrt{x^3} \left(\sqrt{2 + \frac{1}{x^3}} - 1 \right) = +\infty \cdot (\sqrt{2} - 1) = +\infty \end{aligned}$$

$\downarrow$
 0

OTHER

$$\lim_{x \rightarrow +\infty} \sqrt{2x^3+1} - \sqrt{x^3} \cdot \frac{\sqrt{2x^3+1} + \sqrt{x^3}}{\sqrt{2x^3+1} + \sqrt{x^3}} = \lim_{x \rightarrow +\infty} \frac{(\sqrt{2x^3+1})^2 - (\sqrt{x^3})^2}{\sqrt{2x^3+1} + \sqrt{x^3}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{2x^3+1 - x^3}{\sqrt{2x^3+1} + \sqrt{x^3}} = \lim_{x \rightarrow +\infty} \frac{x^3+1}{\sqrt{2x^3+1} + \sqrt{x^3}} = \frac{\infty}{\infty} =$$

$$\lim_{x \rightarrow +\infty} \frac{3x^2}{\frac{1}{2\sqrt{2x^3+1}} + \frac{1}{2\sqrt{x^3}}} = \lim_{x \rightarrow +\infty} \frac{3x^2}{\frac{3x^2}{\sqrt{2x^3+1}} + \frac{3x^2}{2\sqrt{x^3}}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{3x^2}{\frac{3x^2(2\sqrt{x^3} + \sqrt{2x^3+1})}{2\sqrt{x^3}\sqrt{2x^3+1}}} = \lim_{x \rightarrow +\infty} \frac{2\sqrt{x^3}\sqrt{2x^3+1}}{2\sqrt{x^3} + \sqrt{2x^3+1}} \leftarrow \text{INUTILE}$$

$$= \lim_{x \rightarrow +\infty} \frac{3x^2}{3x^2 \left(\frac{1}{\sqrt{2x^3+1}} + \frac{1}{2\sqrt{x^3}} \right)} = \lim_{x \rightarrow +\infty} \frac{1}{\frac{1}{\sqrt{2x^3+1}} + \frac{1}{2\sqrt{x^3}}} = \frac{1}{\frac{1}{\infty} + \frac{1}{\infty}} =$$

$$= \frac{1}{0+0} = \frac{1}{0} = +\infty$$

$$y = 3\sqrt[3]{x^2-8} + 2$$

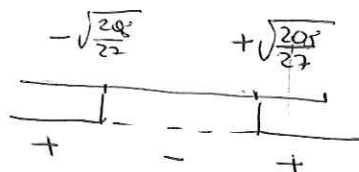
$$D:]-\infty; +\infty[$$

$$(S) \quad 3\sqrt[3]{x^2-8} + 2 \geq 0 \quad \sqrt[3]{x^2-8} \geq -\frac{2}{3} \Rightarrow \text{forma } \sqrt[n]{Ax+B} \geq k \Rightarrow$$

$$\Rightarrow x^2-8 \geq \left(-\frac{2}{3}\right)^3 \Rightarrow x^2-8 \geq -\frac{8}{27} \Rightarrow x^2-8+\frac{8}{27} \geq 0 \Rightarrow$$

$$\Rightarrow x^2 + \frac{-216+8}{27} \geq 0 \Rightarrow x^2 - \frac{208}{27} \geq 0 \Rightarrow x^2 - \frac{208}{27} = 0$$

$$\Rightarrow x = \pm \sqrt{\frac{208}{27}} \approx \pm 2,77$$



(Int.)

$$\text{one } x \quad \begin{cases} y=0 \\ y=3\sqrt[3]{x^2-8}+2 \end{cases} \Rightarrow 3\sqrt[3]{x^2-8}+2=0 \Rightarrow 3\sqrt[3]{x^2-8}=-2 \Rightarrow \sqrt[3]{x^2-8}=-\frac{2}{3} \Rightarrow$$

$$\Rightarrow x^2-8 = \left(-\frac{2}{3}\right)^3 \Rightarrow x^2-8 = -\frac{8}{27} \Rightarrow x^2-8+\frac{8}{27}=0 \Rightarrow x^2-\frac{208}{27}=0$$

$$\Rightarrow x_{1/2} = \pm \sqrt{\frac{208}{27}} \approx \pm 2,77 \quad A\left(-\sqrt{\frac{208}{27}}; 0\right)$$

$$B\left(+\sqrt{\frac{208}{27}}; 0\right)$$

punti int. one x

(si poteva dedurre dallo studio del segno !!)

oss y

$$\begin{cases} x=0 \\ y=3\sqrt[3]{x^2-8}+2 \end{cases} \Rightarrow y = 3\sqrt[3]{0^2-8}+2 = 3\sqrt[3]{-8}+2 = 3\sqrt[3]{-8}+2 = 3\cdot(-2)+2 \Rightarrow$$

$$\Rightarrow y = -6+2 = -4 \Rightarrow C(0; -4) \text{ int. one } y$$

↑ ok poiché per $x=0$ la fun è negativa!

(Lim)

$$\lim_{x \rightarrow -\infty} 3\sqrt[3]{x^2-8}+2 = \lim_{x \rightarrow -\infty} 3\sqrt[3]{(-\infty)^2-8}+2 = +\infty$$

$$\lim_{x \rightarrow +\infty} 3\sqrt[3]{x^2-8}+2 = +\infty$$

N.B. NON CI SONO NE' AS. ORZ. NE AS EVERT!

Ricerca dell'asintoto obliquo

$$m = \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = \lim_{x \rightarrow \pm\infty} \frac{\sqrt[3]{x^2-8} + 2}{x} = \frac{\infty}{\infty}$$

non posso de l'Hopital
poiche' non risolve
l'indeterminazione.

osservo: numeratore ordine $\frac{2}{3}$
denominatore ordine 1 } mi aspetto che vinca il denominatore.
perche' $1 > \frac{2}{3}$

$$m = \lim_{x \rightarrow \pm\infty} \frac{\sqrt[3]{x^2-8} + 2}{x} = \lim_{x \rightarrow \pm\infty} \frac{\sqrt[3]{x^2-8}}{x} + \frac{2}{x} = \lim_{x \rightarrow \pm\infty} \frac{\sqrt[3]{x^2(1-\frac{8}{x^2})}}{x} + \frac{2}{x} =$$

$$= \lim_{x \rightarrow \pm\infty} \frac{3 \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{1-\frac{8}{x^2}}}{x} + \frac{2}{x} = \lim_{x \rightarrow \pm\infty} 3 \frac{x^{2/3}}{x} \cdot \sqrt[3]{1-\frac{8}{x^2}} + \frac{2}{x} =$$

$$= \lim_{x \rightarrow \pm\infty} 3 \cdot x^{\frac{2}{3}-1} \cdot \sqrt[3]{1-\frac{8}{x^2}} + \frac{2}{x} = \lim_{x \rightarrow \pm\infty} 3 \cdot x^{-\frac{1}{3}} \cdot \sqrt[3]{1-\frac{8}{x^2}} + \frac{2}{x} =$$

$$= \lim_{x \rightarrow \pm\infty} \left(\frac{3}{\sqrt[3]{x}} \cdot \sqrt[3]{1-\frac{8}{x^2}} + \frac{2}{x} \right) = \lim_{x \rightarrow \pm\infty} 0 \cdot 1 + 0 = 0 \Rightarrow \text{As. ob.}$$

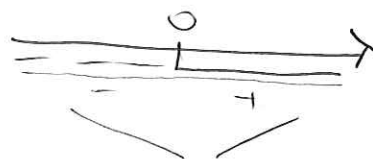
$\begin{matrix} \downarrow & & \downarrow & & \downarrow \\ \pm\infty & & 0 & & 0 \end{matrix}$

DERIVATA PRIMA

$$y' = 3 \cdot \frac{1}{3} (x^2-8)^{\frac{1}{3}-1} \cdot 2x = 2x (x^2-8)^{-\frac{2}{3}} = \frac{2x}{\sqrt[3]{(x^2-8)^2}} > 0$$

$$2x \geq 0 \Rightarrow x \geq 0$$

$$\sqrt[3]{(x^2-8)^2} > 0 \Leftrightarrow (x^2-8)^2 > 0 \quad \forall x \in D$$



OSSERVAZIONE || Per y' non esiste per i valori che annullano
IL SUO DENOM. CHE SONO $\pm\sqrt{8}$ e $\in D$.
LA y' E' DISCONTINUA NEL DOMINIO

Braccio colorato i limiti a destra e sinistra
dei punti di discontinuità della y' ($-\sqrt{8}$ e $+\sqrt{8}$)

$$\lim_{x \rightarrow -\sqrt{8}^-} \frac{2x}{\sqrt[3]{(x^2-8)^2}} = \frac{-2\sqrt{8}}{0} = -\infty \quad (\text{perché la } y' \text{ è negativa a } -\sqrt{8}^-)$$

$$\lim_{x \rightarrow -\sqrt{8}^+} \frac{2x}{\sqrt[3]{(x^2-8)^2}} = \frac{-2\sqrt{8}}{0} = -\infty \quad (\text{perché la } y' \text{ a } -\sqrt{8}^+ \text{ è per le negative})$$

$$\lim_{x \rightarrow \sqrt{8}^-} \frac{2x}{\sqrt[3]{(x^2-8)^2}} = \frac{2\sqrt{8}}{0} = +\infty \quad (\text{perché la } y' \text{ a } +\sqrt{8}^- \text{ è positiva})$$

$$\lim_{x \rightarrow \sqrt{8}^+} \frac{2x}{\sqrt[3]{(x^2-8)^2}} = \frac{2\sqrt{8}}{0} = +\infty \quad (\text{perché la } y' \text{ a } +\sqrt{8}^+ \text{ è positiva})$$

perciò ho che

$$\bullet \lim_{x \rightarrow -\sqrt{8}^-} y' = -\infty \quad \text{e} \quad \lim_{x \rightarrow -\sqrt{8}^+} y' = -\infty \Rightarrow \text{FLESSO A TANG. VERTICALE}$$

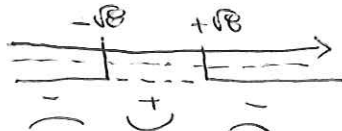
$$\bullet \lim_{x \rightarrow +\sqrt{8}^-} y' = +\infty \quad \text{e} \quad \lim_{x \rightarrow +\sqrt{8}^+} y' = +\infty \Rightarrow \text{FLESSO A TANGENTE VERTICALE}$$

DERIVATA SECONDA

$$y'' = \frac{2 \cdot \sqrt[3]{(x^2-8)^2} - 2x \cdot \frac{2}{3} (x^2-8)^{-\frac{1}{3}} \cdot 2x}{(x^2-8)^{\frac{2}{3} \cdot 2}} = \left(\sqrt[3]{(x^2-8)^2} - \frac{8x^2}{3 \sqrt[3]{x^2-8}} \right) \cdot \frac{1}{\sqrt[3]{(x^2-8)^4}} =$$

$$= \frac{6 \sqrt[3]{(x^2-8)^2} \cdot \sqrt[3]{x^2-8} - 8x^2}{3 \sqrt[3]{x^2-8}} \cdot \frac{1}{\sqrt[3]{(x^2-8)^4}} = \frac{6 \sqrt[3]{(x^2-8)^3} - 8x^2}{3 \sqrt[3]{(x^2-8)^5}} = \frac{6(x^2-8) - 8x^2}{3 \sqrt[3]{(x^2-8)^5}} =$$

$$= \frac{-2x^2 - 48}{3 \sqrt[3]{(x^2-8)^5}} \geq 0 \quad \begin{cases} \text{N. } -2x^2 - 48 \geq 0 \text{ sempre mai } (\Delta < 0) \\ \text{D. } \sqrt[3]{(x^2-8)^5} > 0 \Leftrightarrow (x^2-8)^5 > 0 \Leftrightarrow x^2-8 > 0 \quad x_{1,2} = \pm\sqrt{8} = \pm 2,82 \end{cases}$$



$$y = 3\sqrt[3]{x^2 - 8} + 2$$

